ISO Filter Wavelengths

Version 1.0
SAI/96-240/Dc
18/Dec/96
Andrea Moneti
ISO Calibration Laison

1 Introduction

The purpose of this brief document is to review the transformation of a broad-band (heterochromatic) measurement to a flux density. What is measured by an astronomical instrument is a quantity proportial to the in-band flux

$$E = \int E(\lambda)T(\lambda)d\lambda. \tag{1}$$

where $E(\lambda)$ is the irradiance at the entrance of the instrument (i.e. the telescope), and $T(\lambda)$ is the spectral response of the system.

The transformation involves (i) transforming the in-band flux (W/cm^2) to a flux density ($W/cm^2/\mu m$), by dividing by an appropriate bandwidth, and (ii) determining the correct wavelength to which that flux density should be assigned.

2 The three wavelengths

I will define here the mean, the isophotal, and the effective wavelengths. While the isophotal wavelength is well defined (and little used), the others are often used under different names, hence the need for the definitions.

2.1 The *mean* wavelength

Golay (1974) shows that if $E(\lambda)$ can be adequately represented by a Taylor series truncated after the first derivative, then

$$E = E(\lambda_0) \int_a^b T(\lambda)d\lambda, \tag{2}$$

where

$$\lambda_0 = \frac{\int_a^b \lambda T(\lambda)d\lambda}{\int_a^b T(\lambda)d\lambda} \tag{3}$$

we define as the **mean wavelength**. Note that λ_0 depends solely on the system response.

2.2 The *isophotal* wavelength

The function $E(\lambda)$ does not normally satisfy the condition for expansion in series and for truncation after the first order. It is, however, a continuous function and both $E(\lambda)$ and $T(\lambda)$ remain always positive. One can then use a generalisation of the mean value theorem which states that there exists a $\lambda_{\rm iso}$ such that

$$E = E(\lambda_{\rm iso}) \int_a^b T(\lambda)\mathrm{d}\lambda = \int_a^b E(\lambda)T(\lambda)\mathrm{d}\lambda. \qquad (4)$$

The above equation defines the **isophotal wavelength** $\lambda_{\rm iso}$, which is the wavelength which must be assigned to the monochromatic flux density $E(\lambda_{\rm iso})$ derived from a heterochromatic measurement. Note that determination of $\lambda_{\rm iso}$ requires, strictly speaking, prior knowledge of the shape of $E(\lambda)$.

2.3 The *effective* wavelength

The **effective** wavelength

$$\lambda_{\rm eff} = \frac{\int_a^b \lambda E(\lambda)T(\lambda)\mathrm{d}\lambda}{\int_a^b E(\lambda)T(\lambda)\mathrm{d}\lambda} \qquad (5)$$

is often used in place of $\lambda_{\rm iso}$. It also requires prior knowledge of the shape of $E(\lambda)$, and is more straightforward to compute that $\lambda_{\rm iso}$. Golay notes that $\lambda_{\rm eff}$ is often, but not always, a better approximation of $\lambda_{\rm iso}$ than λ_0.

3 The ISO filter set

Table 1, 2, and 3 list the mean, effective, and isophotal wavelengths, the latter for a $1/\lambda$ spectrum and for Sirius, for all the CAM and PHT filters, and for the IRAS filters. Note that for a $1/\lambda$ spectrum the effective and the isophotal wavelengths are identical, as can be seen by substituting into (4) and (5).

The wavelengths were computed using the PHT filter profiles available in

`/home/pidt2/phtcalib/prog/pro/ibp/spresp/*.dat` on 10-Dec-96, and using the CAM profiles extracted from the calibration files via CIA on 8-May-96. The Sirius model is a special model prepared by Kurucz specifically for the Cohen et al. absolute calibration programme; it can be found in

`/home/sost1/callia/wwwcal/docs/isoprep/cohen/sirius9601.dat.`

Table 1: CAM filter wavelengths

name	λ_0	λ_{iso} [1/λ]	λ_{eff} [Sirius]	λ_{iso} [Sirius]	name	λ_0	λ_{iso} [1/λ]	λ_{eff} [Sirius]	λ_{iso} [Sirius]
sw1	3.59	3.56	3.48	3.53	lw1	4.50	4.48	4.42	4.46
sw2	3.31	3.30	3.30	3.28	lw2	6.83	6.68	6.22	6.46
sw3	4.44	4.41	4.35	4.40	lw3	14.47	14.30	13.83	14.07
sw4	2.78	2.77	2.75	2.75	lw4	6.01	5.99	5.92	5.92
sw5	4.12	4.03	3.77	3.90	lw5	6.79	6.79	6.77	6.75
sw6	3.70	3.70	3.68	3.69	lw6	7.75	7.72	7.64	7.69
sw7	3.05	3.04	3.03	3.03	lw7	9.68	9.61	9.43	9.52
sw8	4.05	4.05	4.05	4.02	lw8	11.33	11.32	11.27	11.28
sw9	3.88	3.87	3.87	3.88	lw9	14.91	14.89	14.82	14.86
sw10	4.63	4.63	4.62	4.63	lw10	11.68	11.35	10.37	10.85
sw11	4.22	4.22	4.22	4.23					

CAM and PHT have two filters “in common”: these are the cosmological gap filter at $\sim 3.6\,\mu$m and the $12\,\mu$m “IRAS” filter; in addition, PHT also has a $25\,\mu$m “IRAS” filter. In practice, the cosmological gap filters of CAM and PHT are very close, while and IRAS $12\,\mu$m filters differ by $0.2\,\mu$m from each other, and are nearly $0.5\,\mu$m longer than the original IRAS filter. The IRAS and the PHT $25\,\mu$m filters are such that their mean wavelengths are similar but their isophotal wavelengths for Sirius differ by nearly $1\,\mu$m; this occurs because the PHT filter weighs more heavily than the IRAS filter the short wavelength part of the profile.

These effects, overall, make the cross calibrations with these filters less trivial than originally thought.

Table 2: CAM filter wavelengths

name	λ_0	λ_{iso} [1/λ]	λ_{eff} [Sirius]	λ_{iso}	name	λ_0	λ_{iso} [1/λ]	λ_{eff} [Sirius]	λ_{iso}
P3.29	3.31	3.30	3.30	3.31	C50	68.70	64.94	57.57	61.00
P3.6	3.59	3.57	3.50	3.54	C60	61.76	60.86	57.63	59.00
P4.85	4.86	4.82	4.70	4.76	C70	80.79	78.55	72.81	76.00
P7.3	7.40	7.26	6.87	7.07	C90	95.29	92.25	81.98	87.00
P7.7	7.66	7.65	7.63	7.66	C100	102.51	100.66	94.91	98.00
P10	10.00	9.97	9.89	9.94	C105	107.15	105.94	101.81	104.17
P11.3	11.35	11.35	11.33	11.29	C120	118.72	116.80	110.04	113.64
P11.5	11.89	11.57	10.62	11.10	C135	155.10	151.71	142.35	147.06
P12.8	12.82	12.78	12.68	12.74	C160	172.33	168.32	155.26	161.29
P16	15.16	15.12	14.99	15.06	C180	181.00	178.80	172.04	175.44
P20	21.03	20.68	19.53	20.10	C200	202.27	200.88	196.51	200.00
P25	23.80	23.52	22.71	23.10					
P60	60.85	59.84	56.22	58.00					
P100	102.46	101.11	96.82	99.00					

Table 3: IRAS filter wavelengths

name	λ_0	λ_{iso} [1/λ]	λ_{eff} [Sirius]	λ_{iso}
A	11.58	11.20	10.08	10.63
B	23.90	23.36	21.72	22.35
C	61.61	59.48	51.97	56.00
D	101.93	100.33	95.28	98.00